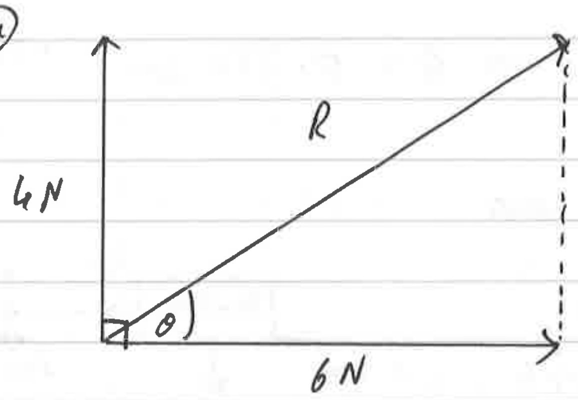
Chapter 4

Exercise 4A (P69)

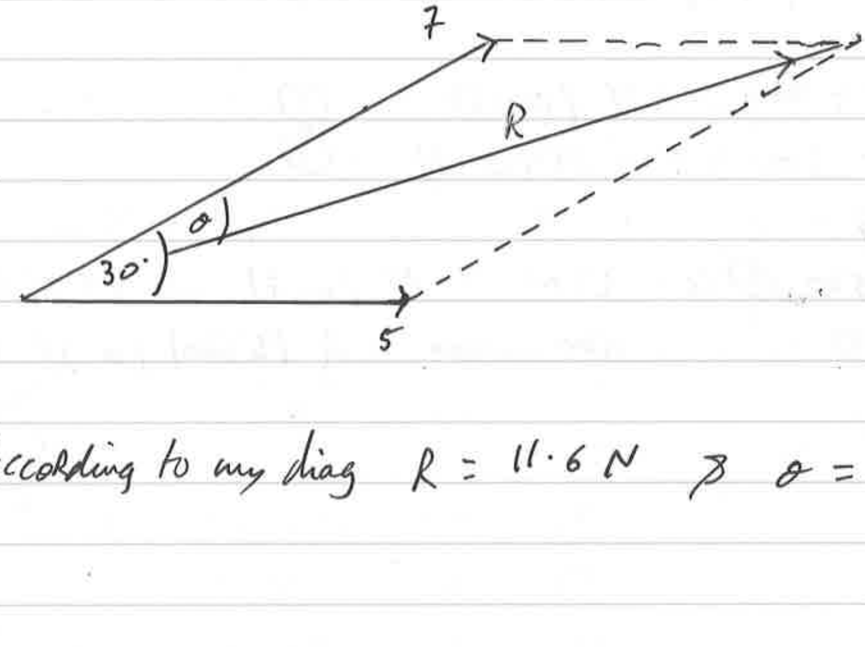
① (a)



By measurement
$R = 7.3\ N$
$\theta = 35°$

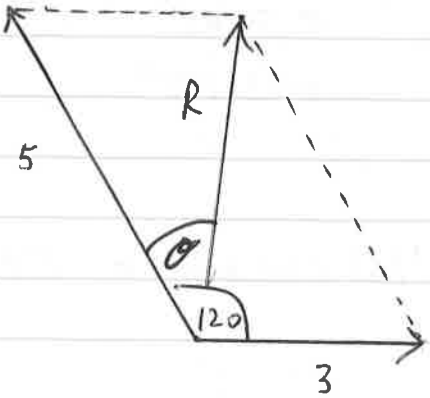
Answer differs from Book due to accuracy of diagram & measurement

(b)



According to my diag $R = 11.6\ N$ & $\theta = 12°$

(c)



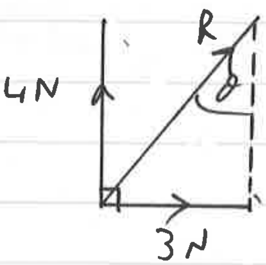
According to my diagram

$R = 4.3\,N$

$\theta = 38^\circ$

Answers differ from book ans due to accuracy of diagram & of Reading From The ".

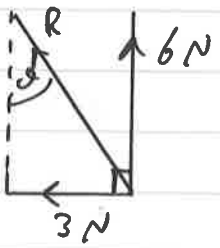
(2) (a)



$R = \sqrt{3^2+4^2} = 5\,N$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$

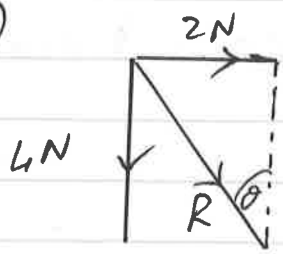
(b)



$R = \sqrt{3^2+6^2} = 6.71\,N$

$\theta = \tan^{-1}\left(\frac{3}{6}\right) = 26.57^\circ$

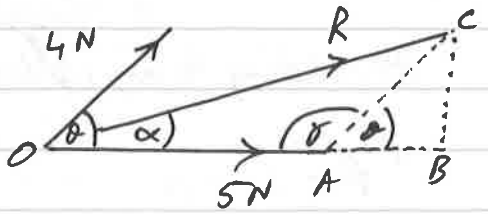
(c)



$R = \sqrt{2^2+4^2} = 4.47N$

$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.57^\circ$

(d)



$\theta = 65^\circ$

$\gamma = 115^\circ$

So $R^2 = 4^2 + 5^2 - 2(4)(5)\cos 115 = 57.91^\circ$

$\Rightarrow R = 7.61\text{ N}$

Now $AC = 4$ & $\angle CAB = 65$ So $CB = 4\sin 65$

& $AB = 4\cos 65$

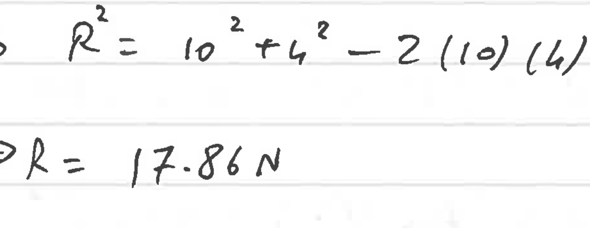
So in $\triangle OBC$

$$\tan\alpha = \frac{4\sin 65}{5 + 4\cos 65} \quad \left(= \frac{CB}{OB} = \frac{CB}{OA + AB} \right)$$

$$= 0.542$$

$\Rightarrow \alpha = 28.45^\circ$

(e)



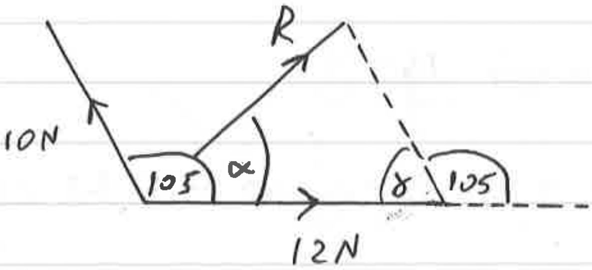
$\gamma = 140^\circ$

So $R^2 = 10^2 + 4^2 - 2(10)(4)\cos 140 = 318.89$

$\Rightarrow R = 17.86\text{ N}$

By Sine Rule: $\dfrac{\sin\alpha}{4} = \dfrac{\sin 140}{17.86} \Rightarrow \alpha = 18.9^\circ$

(f)



$\gamma = 75^\circ$

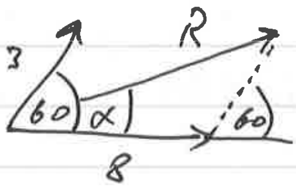
So $R^2 = 12^2 + 10^2 - 2(12)(10)\cos 75 = 181.88$

$\Rightarrow R = 13.49$ N

By Sine Rule : $\dfrac{\sin\alpha}{10} = \dfrac{\sin 75}{13.49}$

$\Rightarrow \alpha = 45.73^\circ$

(3) (a)



$R^2 = 8^2 + 3^2 - 2(8)(3)\cos 120$
$= 97$

$\Rightarrow R = 9.85$ N

By Sine Rule :

$\dfrac{\sin\alpha}{3} = \dfrac{\sin 120}{9.85} \Rightarrow \alpha = 15.3^\circ$

(b) as for (a) but with angle as 50°

$\therefore R^2 = 8^2 + 3^2 - 2(8)(3)\cos 130 = 103.85$

$\Rightarrow R = 10.19$ N

and $\dfrac{\sin\alpha}{3} = \dfrac{\sin 130}{10.19} \Rightarrow \alpha = 13.03^\circ$

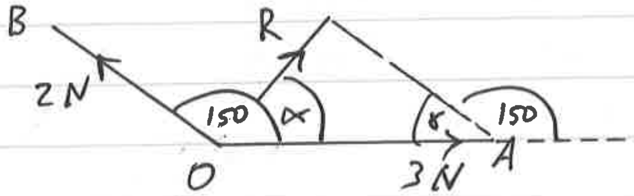
(c) As for (a) but with angle 160°

So $R^2 = 8^2 + 3^2 - 2(8)(3)\cos 20 = 27.89$

$\Rightarrow R = 5.28 \text{ N}$

Sin Rule: $\dfrac{\sin \alpha}{3} = \dfrac{\sin 20}{5.28} \Rightarrow \alpha = 11.2^\circ$

(4)



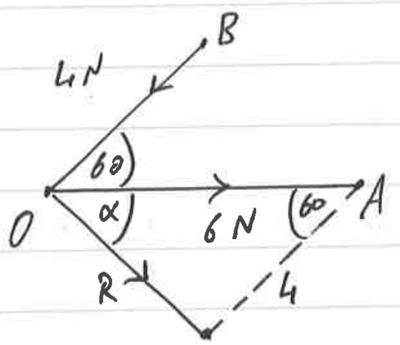
$\gamma = 30^\circ$

So $R^2 = 3^2 + 2^2 - 2(3)(2)\cos 30 = 2.61$

$\Rightarrow R = 1.62 \text{ N}$

By Sin Rule: $\dfrac{\sin \alpha}{2} = \dfrac{\sin 30}{1.62} \Rightarrow \alpha = 38.12^\circ$

(5)



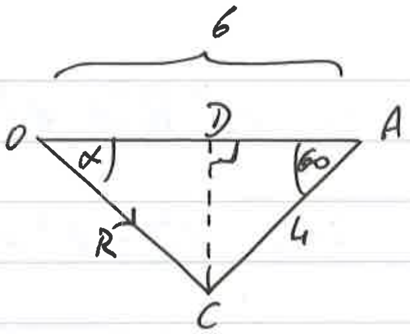
$R^2 = 6^2 + 4^2 - 2(6)(4)\cos 60$

$= 28$

$\Rightarrow R = \sqrt{28} = 5.29 \text{ N}$

Sine Rule: $\dfrac{\sin \alpha}{4} = \dfrac{\sin 60}{5.29} \Rightarrow \alpha = 40.9^\circ$

OR



$CD = 4\sin 60$

$\therefore DA = \sqrt{4^2 - 4^2\sin^2 60}$

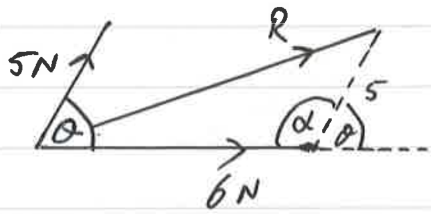
$= 4\cos 60 = 2$

So $OD = 6 - 2 = 4$

$\therefore \tan\alpha = \dfrac{CD}{OD} = \dfrac{4\sin 60}{4} = \sin 60 = \dfrac{\sqrt{3}}{2}$

So $\alpha = 40.9^\circ$

(6)



$R^2 = a^2 + b^2 - 2(a)(b)\cos\alpha$

$\therefore 9^2 = 6^2 + 5^2 - 2(6)(5)\cos\alpha$

$\Rightarrow \alpha = 109.47$

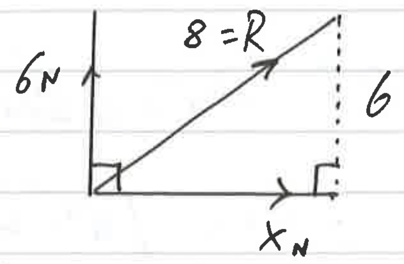
So $\theta = 180 - 109.57 = 70.53^\circ$

(7) As (6) : $8^2 = 10^2 + 4^2 - 2(10)(4)\cos\alpha$

$\Rightarrow \alpha = 49.46^\circ$

So $\theta = 180 - 49.46^\circ = 130.54^\circ$

(8)

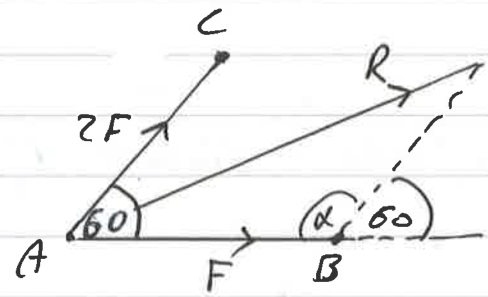


So $R^2 = 6^2 + x^2$

i.e. $8^2 = 6^2 + x^2$

$\Rightarrow \quad x = \sqrt{64-36} = 5.29N$

(9)



$\alpha = 120$

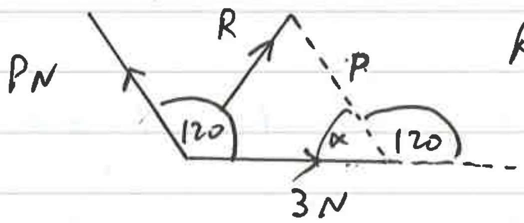
So $R^2 = F^2 + 4F^2 - 2F(2F)\cos 120$

$\Rightarrow \quad R = \sqrt{7}.F$ newtons

<u>Sine Rule</u> : $\dfrac{\sin RAB}{2F} = \dfrac{\sin 120}{\sqrt{7}.F}$

$\Rightarrow$ angle Between R & AB $= 40.89$

(10)



$R = 7 \quad \& \ \alpha = 60^\circ$

So $\quad 7^2 = 3^2 + P^2 - 2(3)(P)\cos 60$

$\therefore 40 = P^2 - 3P \quad \Rightarrow P^2 - 3P - 40 = 0$

$\Rightarrow (P-8)(P+5) = 0$

$\therefore \ P = 8 \ N \quad$ (discard −ve answer)

(11) As (10): given 45°, $\alpha = 135°$

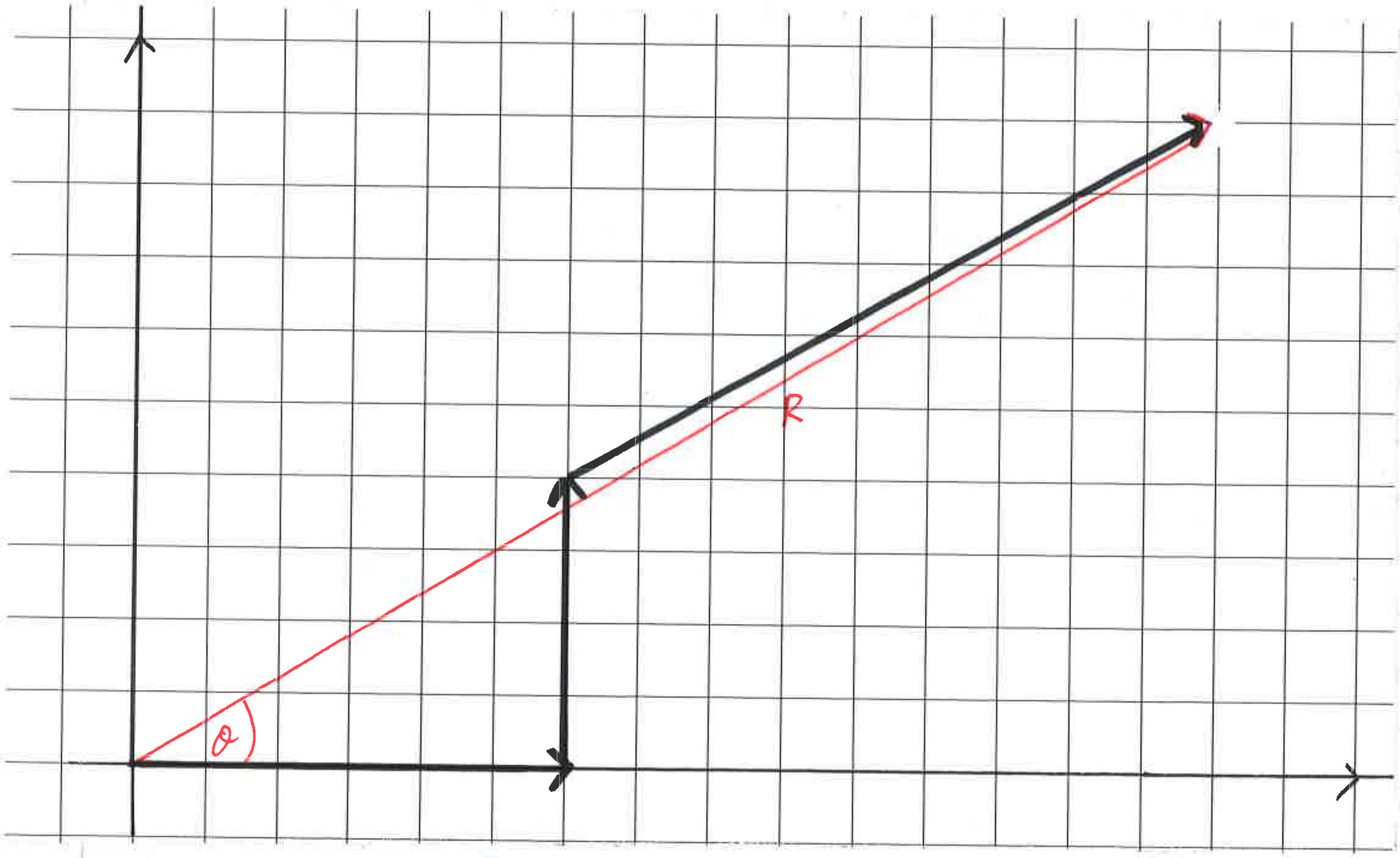
So $15^2 = Q^2 + 8^2 - 2(Q)(8)\cos 135$

$\therefore 161 = Q^2 + 11.31Q \Rightarrow Q^2 + 11.31Q - 161 = 0$

$$\therefore Q = \frac{-11.31 \pm \sqrt{11.31^2 + 4(161)}}{2} = 8.24\text{ N}, -12.58\text{N}$$

So $Q = 8.24$ N.

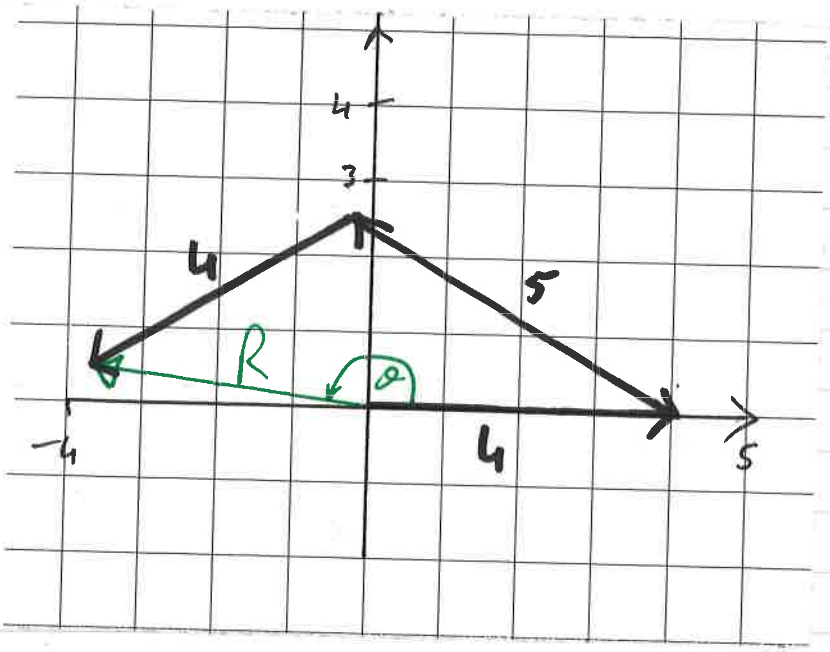
(12) (a)



R = 17.2 cm

θ = 31.5° according to my diagram (≃ 32° allowing for error in drawing)

(b)



R = 3.75 N (to within the limits of accuracy of the diagram)

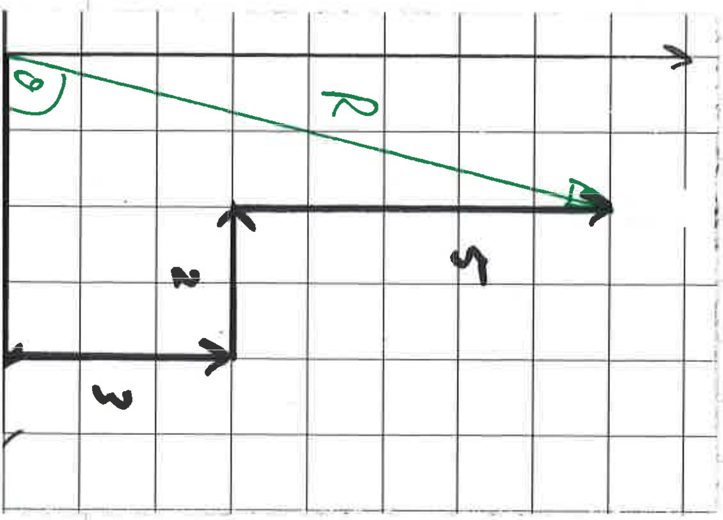
θ = 173° (again to the accuracy given by the diagram)

(c) Left as Exercise

(13) – (14) Left as Exercises.

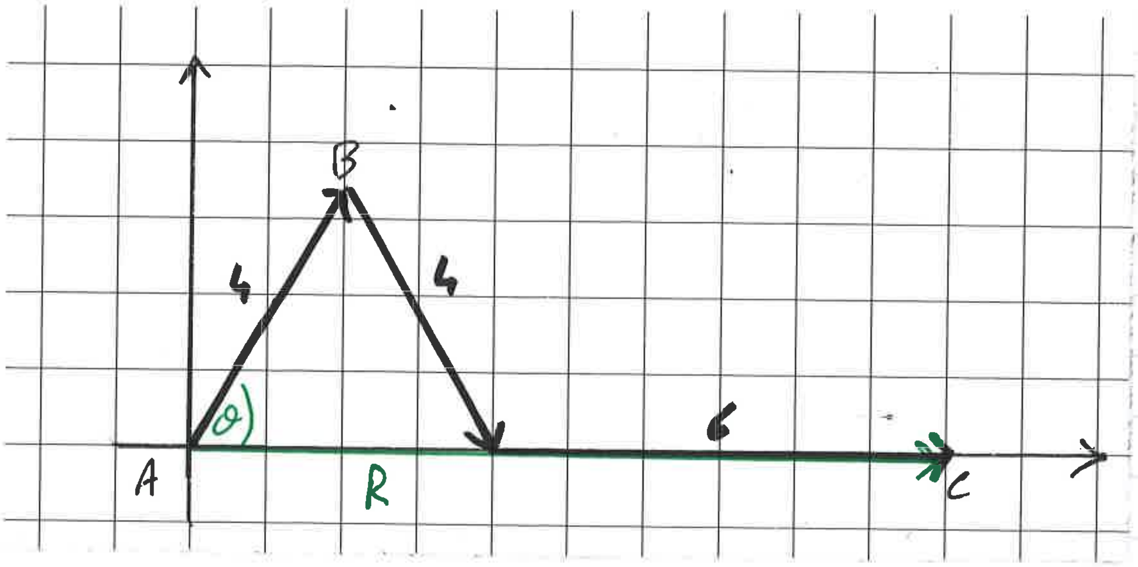
(15) Although the geometry of ABCD is a Square we draw a force diagram (which will Not be square)

Diagram



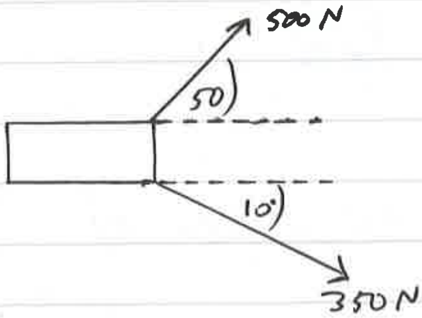
R = 8.3 N
θ = 76°

(16) Although ABC is a triangle The diagram of forces is not.



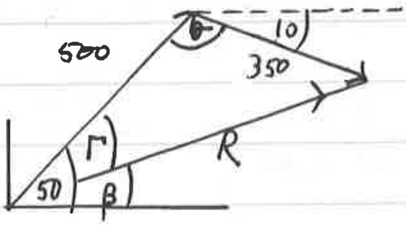
$R = 10\,N$; $\theta = 60°$ (Equilateral Δ)

(17) given

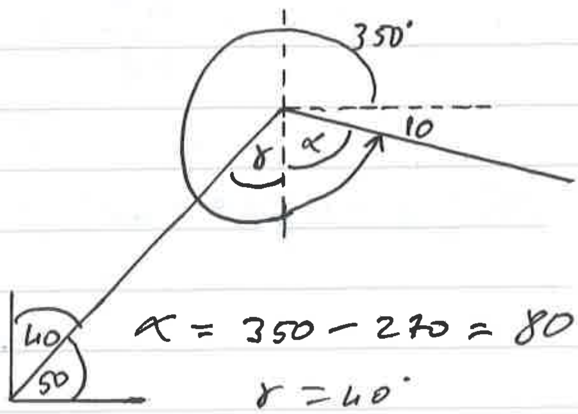


Ropes are Not horizontal But The block moves in a horizontal direction.

So



For angle θ



$\alpha = 350 - 270 = 80°$

$\gamma = 40°$

$\Rightarrow \theta = 40 + 80 = 120°$

So $R^2 = 500^2 + 350^2 - 2(500)(350)\cos 120$

$\Rightarrow R = 740\ N$

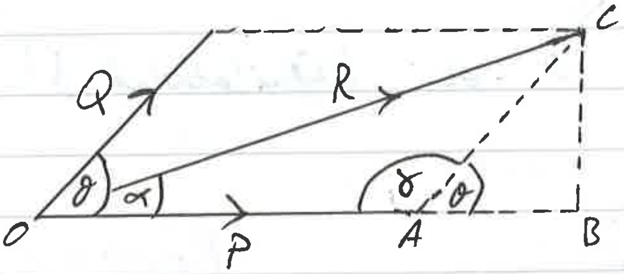
For β : $\beta = 50 - \Gamma$. now use Sine Rule To Find Γ

$\therefore \dfrac{\sin \Gamma}{350} = \dfrac{\sin 120}{740} \Rightarrow \Gamma = 24.18°$

$\therefore \beta = 25.82°$

(18) left as Exercise

(19)



$\gamma = 180 - \theta$

So $R^2 = P^2 + Q^2 - 2PQ\cos(180 - \theta)$

$= P^2 + Q^2 + 2PQ\cos\theta$ ✓

Also $AC = Q$ So $AB = Q\cos\theta$ & $BC = Q\sin\theta$

$\therefore \tan\alpha = \dfrac{BC}{OB} = \dfrac{BC}{OA + AB} = \dfrac{Q\sin\theta}{P + Q\cos\theta}$ ✓

For $P = Q$, $\theta = 40°$: $\tan\alpha = \dfrac{\sin 40}{1 + \cos 40} = 0.36397$

$\Rightarrow \alpha = 20°$